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Abstract Title: Determining Elliptical Conductivity Inclusions via Dipole-Based EIT with Minimal Measurements

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Introduction: Electrical impedance tomography (EIT) traditionally requires numerous boundary measurements to reconstruct conductivity distributions. This work explores a minimal measurement approach using dipole sources to identify small elliptical conductivity anomalies within a unit disc domain.

Objective: Our goal is to determine the minimum number of dipole measurements needed to recover an elliptical anomaly's location, size, and shape parameters, while investigating the stability of this inverse problem and optimal experiment design strategies.

Methods: We derive a linearized forward model relating boundary voltage measurements to anomaly parameters. For a dipole source at angle ϕ, the measured data is the tangential derivative of the perturbational voltage. We derive explicit formulas for the forward map and analyze the Jacobian matrix to assess problem stability. Two approaches for optimal experiment design are presented: (1) a Bayesian framework using Kullback-Leibler divergence to maximize information gain, and (2) a deterministic approach based on Jacobian conditioning.

Results: We show that three dipole measurements suffice to uniquely determine an anomaly's location (b_1, b_2) and area A, with stability estimates provided. Five measurements are needed for full elliptical parameter recovery (axes and orientation), though this proves less stable. Numerical experiments demonstrate successful reconstruction with noise, validating our theoretical findings. The Bayesian and deterministic experiment design approaches yield comparable optimal measurement configurations.

Significance: This work presents a novel EIT paradigm requiring only paired electrode measurements, significantly reducing instrumentation complexity compared to traditional multi-electrode approaches. The theoretical framework provides guidance for practical implementations where minimal measurements are essential, such as in medical imaging or geophysical prospecting.

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